PetaApps Cloud Physics Workshop

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by

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Outline

- Introduction to linear system
- Current approach
- First attempt: Domain Decomposition
- Discussion J, GS, CGS, BICG and GMRES and their limits and performance
- Why GMRES?
- Leveraging the dynamics
- GMRES with recycling
- GMRES with preconditioner
- Cauchy contour approach with GMRES

Intro: A model flow for a droplet in a turbulent cloud

$$u(x,t) = \sum_{k=1}^{N_p} u_s(r^{(k)}; a^{(k)}, V^{(k)} - U(Y^{(k)}, t) - u^{(k)})$$

$$u_s(\vec{r}^{(k)}; a^{(k)}, V_p^{(k)}) = \frac{3}{4}\left[\frac{a^{(k)}}{r^{(k)}} - \left(\frac{a^{(k)}}{r^{(k)}}\right)^3\right]\frac{\vec{r}^{(k)}}{(r^{(k)})^2}(V_p^{(k)} \cdot \vec{r}^{(k)}) + \left[\frac{3}{4}\frac{a^{(k)}}{r^{(k)}} + \frac{1}{4}\left(\frac{a^{(k)}}{r^{(k)}}\right)^3\right]V_p^{(k)}$$

Unknown ↓

$$u^{(k)} = \sum_{m=1, m\neq k}^{N_p} u_s\left(d^{(mk)}; a^{(m)}, V^{(m)} - U(Y^{(m)}, t) - u^{(m)}\right), k = 1 : N_p$$

+ 50-times radius truncation

Unknown ↗

Current approach

- Mixture of Jacobi and Gauss-Seidel
  - Local Gauss-Seidel
  - Global Block-Jacobi
- Advantages
  - Easy implementation
  - Working and tested against Matlab
- Disadvantages
  - Starting to show slow convergence

First attempt: Domain Decomposition

- **Idea**: use domain decomposition with overlap to speed up convergence.
- **Why**: since the problem is mainly local(sparse), DD seems suitable.
- **Results**: local convergence achieved but no global convergence for test problems with small number of particles.
- **Conclusions**: Method not suitable since we even want to have more particles which implies more local interactions and overlap zones increase fast in 3D.

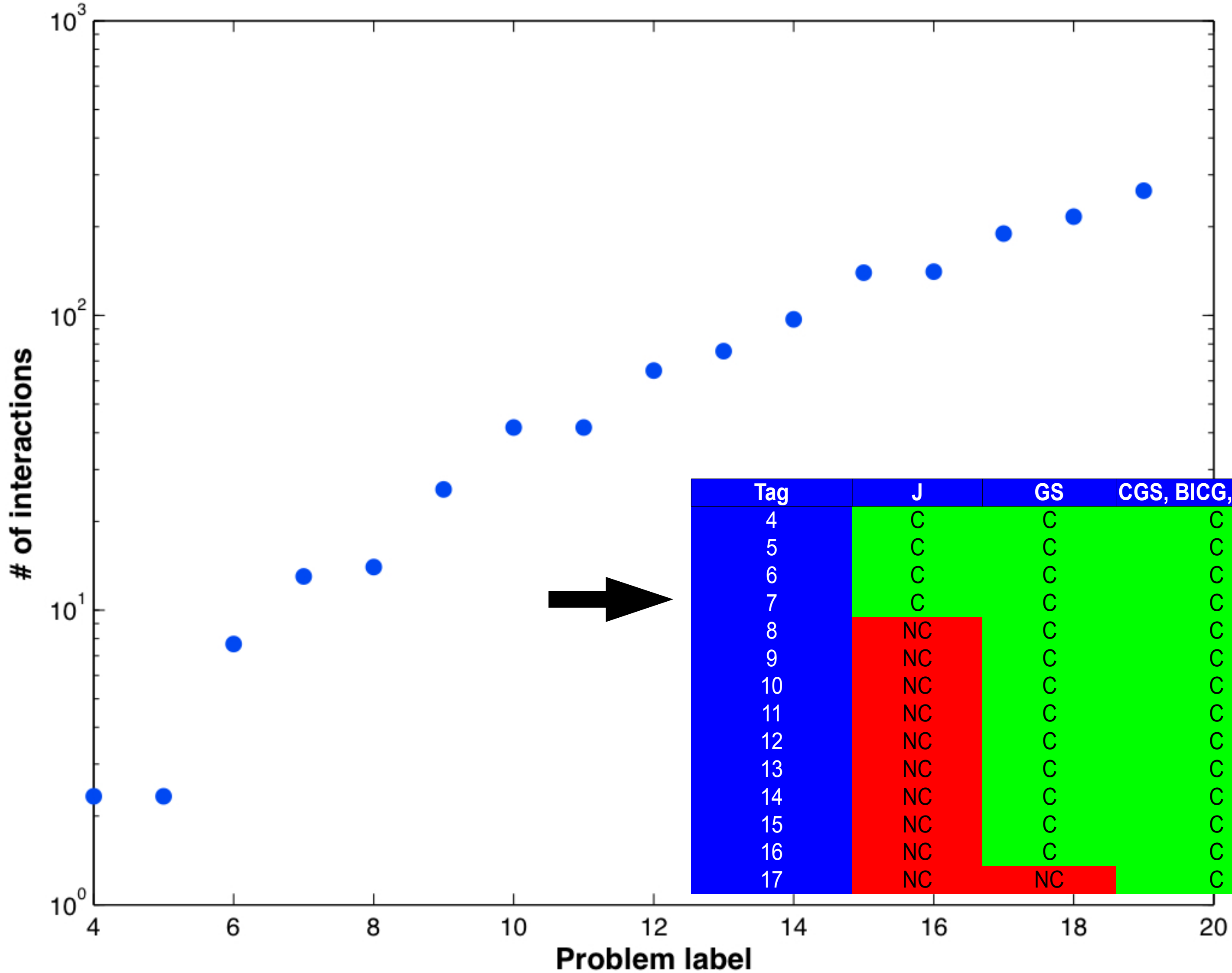
Discussion J, GS, CGS, BICG and GMRES and their limits and performance

- Setup Experiments
  - Small experiments
  - Parameters initialized random
  - We add particles and kept radius fixed

Convergence output

Tag	J	GS	CGS, BICG, GMRES
4	C	C	C
5	C	C	C
6	C	C	C
7	C	C	C
8	NC	C	C
9	NC	C	C
10	NC	C	C
11	NC	C	C
12	NC	C	C
13	NC	C	C
14	NC	C	C
15	NC	C	C
16	NC	C	C
17	NC	NC	C

Convergence output



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12	NC	C	C
13	NC	C	C
14	NC	C	C
15	NC	C	C
16	NC	C	C
17	NC	NC	C

Why GMRES?

- Advantages
  - Matrix-free approach
  - Only require Ax and not A'x
  - Performed better than CGS and BICG in experiments with high # interactions
  - Ax can be parallelize easily
- Disadvantages
  - Memory requirements increase quadratically with iterations(but restart could be use)

Leveraging the dynamics

A and b vary slowly $A^{(i)}x^{(i)} = b^{(i)}$

- Do we expect consecutive solutions to be close?
- Can we take advantage of it?
- Has somebody tried to solve this before?
- Does it work for us?
- Are consecutive Krylov sub-spaces close?

GMRES with recycling

$$A^{(i)}x^{(i)} = b^{(i)}$$

- Do we expect consecutive solutions to be close? **Yes**
- Can we take advantage of it? **Yes**
- Has somebody tried to solve this before? **Yes, GCRO-DR**
- Does it work for us? **Not as expected.**
- Are consecutive Krylov sub-spaces close? **?**

GMRES with preconditioner

- We could have a left, right or both preconditioner, but what would be a good preconditioner?
  - Use same kernel but increase order of decay from 1/r to 1/r^3
    - Advantages: Converges very fast
    - Disadvantage: Still working on how to use it, direct use as preconditioner does not speed up convergence.

Cauchy contour approach with GMRES

- Thid idea has been around for long time
- Strong requirements needed
- Idea: Instead of solving the system directly we solve several 'easier' linear system of equations.
- How:

$$A^{-1}b = \frac{1}{2\pi i}\int_\gamma \frac{1}{z}\left(zI - A\right)^{-1} b\, dz$$

The End

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