Particle Dynamics in Multiphase Flow

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Thanks also to: Kyongmin Yeo (LBNL), Eric Climent (IMFT, France), Don Liu (LA Tech), Sune Lomholt & Lian-Ping Wang
Multiphase Flow in Clouds

Clouds as a mixture of air, water vapor, liquid water and ice

Buoyant convection:
- Temperature
- Water vapor content
- Mass loading of droplets & ice

Local heat sources/sinks for air mass:
- Latent heat of freezing/melting
- Latent heat of condensation/vaporization

Fair weather cumulus clouds, www2010.atmos.uiuc.edu

- Dilute, dispersed multiphase flow: low particle mass loadings of 0.5 – 5 gm/kg
- Very low particle volume fractions, $\phi \sim 10^{-6}$ or so
Byers-Braham model of thunderstorm cell

- **Towering Cumulus Stage**
  - Buoyant thermal, strong updraft and entrainment

- **Mature Stage**
  - Condensing droplets and precipitation creates downdraft

- **Dissipating Stage**
  - Downdrafts, precipitation block further upflow at base

Dynamics of Water Droplets

Droplets form initially by condensation of water vapor:
- Homogeneous nucleation – slow process, high supersaturations
- Heterogeneous nucleation – faster, initiated by ambient aerosol (CCN)
- Continued growth by diffusion and condensation of vapor on droplet – effective up to radius $a \sim 10 \, \mu m$

Water droplet motion in the ambient turbulent flow:
- Dense spherical particles fall relative to the surrounding air mass
- Droplets advected by the large scale turbulent flow, updrafts may counter settling under gravity
- Droplets may have significant inertia

Interaction of droplets:
- Pairwise interactions of droplets can lead to collisions and coalescence
- Primary growth mechanism for droplets to form precipitation

Mass loading of the droplets (liquid water) can modify the bulk flow: directly or indirectly by cycle of condensation and re-evaporation

Topics

- Settling rates of isolated droplets or spherical particles
- Effects of particle inertia on motion of small particles:
  - inertial bias
  - settling rates
- Non-spherical particles
- Interactive motion of particles
  - Simulation methods
  - Stokes flow
  - Finite Re effects
  - Collective motion
Settling of isolated water droplets

Terminal settling velocity, $W$

$$m_p g = 6\pi \mu a W, \quad m_p = \frac{4}{3} \rho_p \pi a^3$$  \hspace{1cm} (Stokes)

$$\text{Re}_p = \frac{2aW}{\nu} \ll 1$$

Estimate for small particles, where $\text{Re}_p \propto a^3$

Larger drops:

$$m_p g = \frac{1}{2} C_D \pi a^2 \rho_0 W^2$$  \hspace{1cm} (Inertial scaling)

Estimate for large particles, $C_D \sim 0.45$: $\text{Re}_p \propto a^{3/2}$

Can define a force Reynolds number: $\text{Re}_F \equiv m_p g / \mu \nu = 3\pi \text{Re}_p$  \hspace{1cm} (Stokes)

<table>
<thead>
<tr>
<th>Radius $a$</th>
<th>10 $\mu$m</th>
<th>23.5 $\mu$m</th>
<th>32.3 $\mu$m</th>
<th>41.5 $\mu$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$ cm/sec</td>
<td>1.20</td>
<td>6.4</td>
<td>11.6</td>
<td>18.1</td>
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<tr>
<td>$\text{Re}_F$</td>
<td>0.151</td>
<td>1.95</td>
<td>5.09</td>
<td>10.76</td>
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<tr>
<td>$\text{Re}_P$</td>
<td>0.016</td>
<td>0.2</td>
<td>0.5</td>
<td>1.0</td>
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<tr>
<td>$(1+f)$</td>
<td>1.00</td>
<td>1.037</td>
<td>1.079</td>
<td>1.142</td>
</tr>
</tbody>
</table>

In dry air at 20$^\circ$C

Chang & Maxey, JFM 277 (1994)
Rigid sphere, isolated water droplets?

\[ \mu_{Water} = 1.00 \times 10^{-3} \text{ kg m}^{-1}\text{s}^{-1} \]
\[ \mu_{Air} = 1.81 \times 10^{-5} \quad \mu_{Water}/\mu_{Air} = 55 \]

As a droplet falls, an internal circulation is set up. Continuity of shear stress gives:

\[ \frac{\mu W}{\alpha} \sim \frac{\mu_{Water} U_{Int}}{\alpha} \]
\[ U_{Int} \sim 0.02 W \]

Droplet responds as a rigid sphere but internal circulation is relevant to heat transfer inside the droplet (and lift)

Effective no-slip surface due to large viscosity ratio, at 20°C:

Streamlines for \( Re_p = 0.35 \); viscosity ratio = 3

Oliver & Chung (1985) JFM 154, 215
Spherical droplets

For small droplets: Surface tension dominates pressure variation due to flow past particle

\[ \Delta p = \frac{2\gamma}{a} \quad \text{Surface tension} \]

\[ \Delta p = \mu \frac{W}{a} \quad \text{Viscous stress} \]

\[ Ca = \frac{\mu W}{\gamma} \]

Droplet
\[ a = 50 \, \mu m \]
\[ Ca < 0.6 \times 10^{-4} \]

\[ We = \frac{2\rho a W^2}{\gamma} \]

Water drop data from Beard (1976). Figure from Loth Int. J Multiphase Flow (2008)
Turbulence scales

Bulk scales of convection and overturning in cloud

- $L \sim 100 \text{ m}$
- $u' \sim 1 \text{ m/sec}$
- $Re_L \sim 10^7$

Viscous dissipation balances turbulence kinetic energy production:

$$\varepsilon \sim \frac{(u')^3}{L}$$
$$\varepsilon = 15 \nu u'^2 / \lambda^2$$

For atmospheric clouds:

$$\varepsilon = 10 - 10^3 \text{ cm}^2 \text{ s}^{-3}$$

Note: Smallest eddies are usually

$$\sim 3 - 5 \eta_K$$

Scale of viscous dissipation, Kolmogorov length

$$\eta_K = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$$
$$\eta_K \sim 1 \text{ mm}$$
Solid Particles: Droplets in air

Negligible inertia – slow variations in local flow

Quasi-steady balance of drag force and force of gravity:

Terminal fall velocity, $W_s$

$$\frac{dX}{dt} \equiv V(t) = u(X,t) + W_s$$

Applies to low or high $Re_p$

Effects of particle inertia:

$$m_p \frac{dV}{dt} = 6\pi a \mu (u(X,t) - V) + m_p g$$

$$W_s = m_p g / 6\pi a \mu$$

Particle response time,

$$\tau_p = m_p / 6\pi a \mu \quad \& \quad St = \tau_p / \tau_K$$

Balance of inertia with quasi-steady Stokes drag force, based on a “slip velocity”, and the force of gravity.
General unsteady flow with moving particle

Ambient flow $u(x,t)$ without particle and incompressible flow $v(x,t)$ with particle in place but same far field boundary conditions

$$\rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = \rho g - \nabla p + \mu \nabla^2 v$$

$$v = \Omega + x \times \left[ Y - (t) \right] \text{ on sphere}$$

$$v = u(x,t) \text{ as } |x - Y(t)| \to \infty$$

Approximate results for unsteady Stokes flow

Small rigid spherical particle: $Re_p = Wd/\nu << 1$ and $Re_\Gamma = a^2 \Gamma/\nu << 1$, for local velocity gradient $\Gamma \sim U/L$
Approximate result from unsteady Stokes flow

- Change to frame of reference moving with particle velocity: \( z = x - Y(t) \) and \( w = v - V(t) \)
- Split \( w \) into base flow \( w^{(0)} = u - V \) and local disturbance flow \( w^{(1)} \)
- Solve for \( w^{(1)} \) as an unsteady Stokes flow
- Forces due to ambient base flow and forces from disturbance flow – evaluate latter from a generalized Reciprocal Theorem

\[
\begin{align*}
    m_p \frac{dV}{dt} &= (m_p - m_F)g + m_F \frac{Du}{Dt} - \frac{m_F}{2} \frac{d}{dt} \left\{ V - u - \frac{1}{10} a^2 \nabla^2 u \right\} \\
    -6\pi a \mu \left\{ Q + a \int_0^t \frac{dQ}{d\tau} \left[ \pi v(t - \tau) \right]^{-1/2} d\tau \right\}
\end{align*}
\]

\[
Q = V(t) - u(Y(t), t) - \frac{1}{6} a^2 \nabla^2 u \quad \text{(Includes Faxen corrections to drag)}
\]

Assumes initial condition that \( Q(t=0) = 0 \)

From: Maxey & Riley (1983), Gatignol (1983)
Particles in Turbulence

Require, small particle, relative to Kolmogrov scales: \( a < \eta_K \)

Reynolds numbers \( \text{Re}_p \) and \( \text{Re}_\Gamma \) are small:

\[
\left( \frac{a}{\eta_K} \right) \left( \frac{W_0}{v_K} \right) \ll 1; \quad \left( \frac{a}{\eta_K} \right)^2 \ll 1
\]

Kolmogorov velocity scale:

\[ v_K = (\varepsilon \nu)^{1/4} \sim 2 \text{ cm/s} \quad (\varepsilon = 100 \text{ cm}^2\text{s}^{-3}) \]

Heavy particles, gas-solid flows: \( m_p >> m_F \)

- Ignore force due to base flow acceleration
- Ignore added mass force
- Unsteady (Basset) viscous term is usually small:
  
  Compare time for vorticity diffusion and response time

\[
\frac{a^2}{v} = \left( \frac{a}{\eta_K} \right)^2 \tau_K \quad \text{to} \quad \tau_P; \quad \text{note Stokes number} \quad St = \frac{\tau_P}{\tau_K}
\]
Unsteady viscous Stokes (history) forces

Oscillatory motion of a sphere, with motion following an elliptic path:

External force $\mathbf{G}(t)$ on the particle required to produce this motion:

\[
\hat{\mathbf{G}} = \left( m_p + \frac{1}{2} m_F \right) i\sigma \hat{\mathbf{V}} + 6\pi a \mu (1 + \delta e^{i\pi/4}) \hat{\mathbf{V}}
\]

\[
\delta^2 = \frac{\sigma a^2}{\nu} \quad \text{Acceleration } \propto \sigma V
\]

At lower frequencies, history force is more important. Gives a phase shift

In unsteady flow, viscous drag force is not necessarily parallel to velocity.
Steady flow past a sphere: $\text{Re} = \frac{Vd}{\nu}$

Streamline and vorticity contours for $\text{Re} = 0.1$, 10 and 40. Flow is left to right.

Separated flow for $\text{Re} > 20$.

Chang & Maxey, JFM 277 (1994)
Oscillating flow past a sphere

Oscillating flow $U(t) = -A \sigma \sin \sigma t$, shown for $a/A=0.625$ and $Re = 16.7$, over half a cycle at $\Phi = \pi/16$, $\pi/4$, $\pi/2$, $3\pi/4$, $15\pi/16$ and $\pi$. Positive and negative (dots) contours

Chang & Maxey, JFM 277 (1994)
History Term at Finite Reynolds Number


Unsteady Stokes flow, impulsive start of sphere with velocity $V(t)=0$ ($t<0$) and $=V$ ($t>0$):

$$F(t') = 6 \pi a \mu \left( H(t') + \frac{1}{3} \delta(t') + (\pi t')^{-1/2} \right) V \text{ with } t' = a^2 t / \nu$$

At finite Reynolds numbers the unsteady viscous term decays more rapidly in the long term. Still varies as $t^{-1/2}$ in short term.

$$\Delta F \sim t^{-2} \quad \text{start from rest}$$
$$\Delta F \sim t^{-1} \quad \text{stop with a prior wake flow}$$
$$\Delta F \sim t^{-5/2} \exp(-\alpha t) \quad \text{small increase in } V$$
$$\Delta F \sim t^{-2} \exp(-\beta t) \quad \text{larger increase in } V$$

Hinch (1993) JFM 256, 601-603
Steady Cellular Flow – Taylor Green Vortex

2-D Incompressible flow given by a streamfunction:

\[ \psi(x_1, x_2) = U_0 L \sin \left( \frac{x_1}{L} \right) \sin \left( \frac{x_2}{L} \right) \]

Use \( U_0 \) and \( L \) as reference scales so that the flow is

\[ u_1 = \frac{\partial \psi}{\partial x_2} = \sin(x_1) \cos(x_2) \quad u_2 = -\frac{\partial \psi}{\partial x_1} = -\cos(x_1) \sin(x_2) \]

Conservation of particle number density or concentration \( C(x, t) \)

For Lagrangian fluid tracer particles

\[ \frac{\partial C}{\partial t} + \nabla \cdot (uC) = 0 \]

If \( \nabla \cdot u = 0 \), \( C(x, 0) = C_0 \), then \( C(x, t) = C_0 \)
Steady Cellular Flow – No inertia

Stommel (1949) J. Marine Res. 8, 24-29 “Trajectories of small bodies sinking slowly through convection cells”

\[ \mathbf{V}(t) = \mathbf{u}(\mathbf{Y}(t), t) + \mathbf{W} \]
Steady Cellular Flow – Gas-solid particles

\[
m_p \frac{dN}{dt} = 6\pi a \mu (u(X,t) - V) + m_p g \quad \text{or} \quad \text{St} \frac{dV}{dt} = (u(Y,t) - V) + W
\]

W=0.25 and St=0.2

W=0.5 and St=0.2

Steady Cellular Flow – Gas-solid particles

“Order versus chaos”

W=0.5 and St=0.2

W=0.25 and St=10

Tilted cell
Steady Cellular Flow – Gas-solid particles

Average over different orientations of cells relative to vertical

![Graph showing average settling velocity](image)

- Large inertia
- Small inertia

\[ A = \frac{1}{St} \]
Low Inertia Approximation: $0 < St << 1$

\[
St \frac{dV}{dt} = (u(Y, t) - V) + W
\]

\[
V(t) = v(Y, t) = (u(Y, t) + W) - St \left( \frac{\partial u}{\partial t} + (u(Y, t) + W) \cdot \nabla u \right)
\]

Effective particle velocity field, $v(x,t)$

“Compressible”

\[
\nabla \cdot v = -St \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}
\]

Inertial bias to regions of high strain-rate or low vorticity.

“Heavy particles do not turn corners easily”

\[
= -\frac{1}{4} St \left\{ \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 - \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)^2 \right\}
\]

Strain-rate, $S^2$  Vorticity, $\Omega^2$

Maxey & Corrsin (1986), Maxey (1987a,b)
Particles in locally isotropic turbulence

Forced, stationary isotropic turbulence

Local $C$  \hspace{1cm} Vorticity $\Omega$

$t = 0$

$t = 8 \tau_K$

$\text{Re}_\lambda = 31$, $48^3$ grid

$\tau_P = \tau_K$, $W = \nu_K$

Conditionally averaged particle concentration $\langle C \rangle$

$W = \nu_K, \tau_P = \tau_K, \text{Re}_\lambda = 43$

Wang & Maxey (1993) JFM 256

Strain rate, $S$

Vorticity, $\Omega$
Increased Particle Settling in Turbulence

Preferential Sweeping Mechanism

\[ \langle \Delta V_1 \rangle / \nu_K \]

\[ W = \nu_K \]

\[ \tau_P / \tau_K \]

\[ \langle \Delta V_1 \rangle / u' \]

\[ St = \tau_P / \tau_K \]

Compare also

\[ \tau_P \quad \& \quad \eta_K / W = \tau_K \frac{\nu_K}{W} \]

Eddy-crossing time

Simulation data

32^3 and Re_\lambda = 21;

48^3 and Re_\lambda = 31;

64^3 and Re_\lambda = 43;

96^3 and Re_\lambda = 62,

Wang & Maxey (1993),
J. Fluid Mech. 256
Experimental Observations

Particle dispersion in a turbulent shear layer, Lazaro & Lasheras (1989) – preferential concentration of particles at edges of vortex structure.

Particle concentration on center-plane of a turbulent channel flow, 25 μm glass beads, Fessler & Eaton (1994)
Experiments by Aliseda et al (2002), JFM 468

Water droplet diameters

Re$_{\lambda} = 88 - 48$

\( \langle \Delta V \rangle \)

\( u' \)

Volume fraction \( \alpha \)

\[ W / v_K \]

<table>
<thead>
<tr>
<th>( x ) cm</th>
<th>( u' ) cm/s</th>
<th>( \varepsilon ) m$^2$ s$^{-3}$</th>
<th>L mm</th>
<th>( \lambda ) mm</th>
<th>( \eta ) mm</th>
<th>( \tau_K ) ms</th>
<th>( v_K ) cm/s</th>
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<tbody>
<tr>
<td>83</td>
<td>26.2</td>
<td>1.75</td>
<td>37.7</td>
<td>5.06</td>
<td>0.21</td>
<td>2.92</td>
<td>7.16</td>
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<tr>
<td>207</td>
<td>15.6</td>
<td>0.61</td>
<td>56.2</td>
<td>4.66</td>
<td>0.27</td>
<td>4.96</td>
<td>5.50</td>
</tr>
</tbody>
</table>
Cluster-induced settling

Local accumulation of particles amplifies small ambient mass loading

\[ \frac{\langle \Delta V \rangle}{u'} \]

St = 0.85, 1.38

Volume fraction \( \alpha \)

Loosely clustered particles create local downflow, entrains particles

Cluster size vs threshold

\[ 15\eta_K \quad C / C_0 > 2.2 \]

\[ 7\eta_K \quad C / C_0 > 3.4 \]

Data at \( x = 100 \text{ cm} \)
Non-spherical particles - Ice

Graupel – formed by fast freezing of droplets on ice particle.

Translation motion and rotation of particles are now linked:
- Fall velocity – not vertical
- Rotation by turbulence enhance dispersion

Rotation with vorticity and to align with rate of strain

emu.arsusda.gov/snowsite/rimegraupel/rg.html
Ellipsoids and Spheroids

Stokes particle, no inertia: fluid force and torque

\[ F_i = \mu K_{ij} \left( u_j (Y(t),t) - V_j \right) \]
\[ T_i = \mu R_{ij} \left( \frac{1}{2} \omega_j - \Omega_j \right) + \mu D_{ijk} E_{jk} \]

Axisymmetric spheroid, axis vector \( m \)

\[ V(t) = u(Y(t),t) + W_1 (\hat{g} \cdot m) m + W_2 \left( \hat{g} - \{ \hat{g} \cdot m \} m \right) \]

\[ \frac{dm}{dt} \Omega(t) \times \Omega(t) = \frac{1}{2} \omega(Y(t),t) + Dm \times (E \cdot m) \]

For a prolate spheroid, aspect ratio \( \lambda > 1 \), \( W_1 > W_2 \) and \( D = \left( \frac{\lambda^2 - 1}{\lambda^2 + 1} \right) \)

Falls faster along body axis, rotation is coupled to vorticity and strain-rate \( E \), with particle tending to align with strain-rate axes. Jeffery (1922) orbits for a spheroid in uniform shear flow – regular periodic motion
Steady Cellular Flow – Spheroids

$\lambda = 2$ and $V_\infty = 0.75$

$\lambda = 10$ and $V_\infty = 0.75$

Figure 9. The trajectory of a spheroidal particle settling through the cellular flow showing the path $(X_1, X_2)$ and particle orientation $m = (\cos \theta, \sin \theta)$: (a) aspect ratio $\lambda = 2$, $V_\infty = 0.75$; (b) $\lambda = 10$, $V_\infty = 0.75$. Boxes drawn to mark unit cells.

Steady Cellular Flow – Spheroids

Poincare sections for $\sin \theta = 0$; $D = 0$ and $W_1=0.5$, $W_2=0.4$
Effect of vorticity coupling for rotation but not strain-rate

Chaotic & regular motion
Inertia reduces chaotic range

Non-spherical particle motion
in turbulence largely still to be explored

Work on fiber suspensions –
Saintillan (2005, 2006)

Interactions of Particles in Flows

Interactions of pairs of particles

Stokes flow

Finite Reynolds number effects

- Experimental observations
- Numerical simulations to solve full Navier-Stokes equations

Match boundary conditions on flow

Match boundary conditions on particle surface, sphere radius $a$: $\mathbf{u} = \mathbf{\Omega} + \mathbf{x} - \mathbf{X}$( )

Evaluate fluid forces, torques on particles and move particles

**Numerical Schemes**: Finite Element Method – ALE with moving mesh (Hu), Distributed Langrange Multiplier DLM (Glowinski, Patankar), PHYSALIS (Prosperetti), LBM, Stokesian Dynamics (Brady), Immersed Boundary Methods (Peskin, Mittal & Iaccano (2005), Uhlman (2005))
Force-Coupling Method: Flow

- Low-order force multipole expansion to represent the particles
- Replace Dirac delta function by Gaussian distributions

\[ \Delta(x - Y) = (2\pi\sigma^2)^{-3/2} \exp\left(\frac{(x - Y)^2}{2\sigma^2}\right) \]

- Incompressible volumetric flow field \( u(x,t) \)
- Exploit self-induced flow inside each particle

\[
\rho_F \left[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = -\frac{\partial p}{\partial x_i} + \mu \nabla^2 u_i + \\
\sum_{n} F_{i}^{n} \Delta(x - Y^n) + \sum_{n} G_{ij}^{n} \frac{\partial \Theta}{\partial x_j} (x - Y^n)
\]

**Force Monopole**
\[ a = \sigma_\Delta \sqrt{\pi} \]

**Force Dipole**
\[ a = \sigma_\Theta \left(6\sqrt{\pi}\right)^{1/3} \]

Particle Phase Motion

Particle position $Y(t)$, velocity $V(t)$ and angular velocity $\Omega$ are evaluated in terms of fluid velocity and vorticity $\omega$:

$$\frac{dY}{dt} = V(t) = \int u(x, t) \Delta(x - Y(t)) d^3x$$

$$\Omega_i(t) = \frac{1}{2} \int \omega_i(x, t) \Theta(x - Y(t)) d^3x$$

Force monopole

$$F = F^{(Ext)} + (m_F - m_P) \frac{dN}{dt} + F^{(C)}$$

- Fluid and particles considered as one system – specify a mobility problem for particle motion, i.e. specify force and evaluate velocity.

- Force monopole is force of particle on the fluid needed to move equivalent volume of fluid at the same velocity as the rigid particle.

- Do not impose no-slip condition on particle, integral constraint only.

- Use short-range force barrier for collisions, maintain void fraction.
Force Dipole Coefficients $G$

Symmetric stresslet $G^S$ set so as to ensure volume-averaged rate of strain for each particle is zero

$$\int \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \Theta(x - Y(t))d^3x = 0$$

Stresslet does no work in the KE budget, and acts even for neutrally buoyant particles.

Anti-symmetric $G^A$ set by external torque $T^{(Ext)}$ and by the moments of inertia $I$ for each particle

$$G^A_{ij} = \frac{1}{2} \epsilon_{ijk} T_k$$

where

$$T = T^{Ext} + (I_F - I_P) \frac{d\Omega}{dt}$$

Moment of inertia coefficient $I_F$ chosen suitably
Identical pair of Stokes particles

**Equal Forces**
Faster settling of particle pair

**Opposite forces**
Strong viscous forces resist motion at short range

Comparison of FCM results: ........, monopole; ______, monopole & dipole; , exact [from Batchelor (1972, 1976) JFM 52 & 74]. Need lubrication forces at short-range. Normal force ~ 1/(gap width); tangential force ~ - log (gapwidth)
Two Particles in linear shear flow

Interactions of particles in Stokes flow are reversible. Expect symmetric motion left-right in shear flow. Symmetry broken by effects of surface roughness – contact forces

Irreversible offsets due to contact forces: $R_{\text{ref}} = 2.001a$

<table>
<thead>
<tr>
<th>Entry- left</th>
<th>Exit - right</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y^T/a$</td>
<td>0.2 0.4 0.6 0.7 0.8</td>
</tr>
<tr>
<td>$\Delta y^F/a$</td>
<td>0.631 0.631 0.631 0.700 0.800</td>
</tr>
</tbody>
</table>

Contact only if gap is 0.001a

Two Particles: Drafting, Kissing and Tumbling

Irreversible effects of finite Reynolds number flow

Side and front views of particles falling in a vertical channel

Channel (duct): \(-2 < x, y < 2 \& 0 < z < 16\)
Particle radius \(a = 0.2\) and \(\mu = 1\)

\[
W_{\text{Stokes}} = \frac{F}{6\pi a \mu} = 3.33
\]

\[
\text{Re}_F = 12.6
\]

\[
\text{Re}_p(\text{max}) = 2
\]

DKT : Fortes, Joseph & Lundgren (1987)
JFM 177, 467-483

See ebook by D.D. Joseph at www.efluids.com/e fluids/books

D. Liu, PhD thesis 2004

Video: www2.latech.edu/~donliu/Movies/2.mpg
Experimental Setup (Lomholt et al., IJMF 2002)

- **Fluid:** Glycerol + Water for low Re
- **Spheres of radius 1 mm**
- **Buoyant particles move upwards**

- **Two views:** $x_1-x_2$ & $x_1-x_3$ (mirror)
- **Re (Stokes) = 1.55**
Lift force due to steady shear: Rigid spheres

Saffman, McLaughlin lift force at Re < 1 – wall corrections


Spherical droplet

\[ C_L = \frac{F_L}{\frac{1}{2} \pi \rho a^2 U_C^2} \]

\[ U = 1 + \alpha y \]

Overlap at low shear rates

Sugioka & Komori (2007) JFM 570
Have mentioned some processes related to particle/droplet motion – many issues not discussed:

- How does motion of the droplets in local surroundings influence diffusion of vapor and condensation? Issue of heat and mass transfer locally, with internal circulation within the drop, at Re > 0.
- How to include processes that depend on sub-grid dynamics in an LES model
- Droplet collisions and experiments – coalescence or oscillation and break-up. Charge effects. Related work in combustion sprays and spray cooling systems.

Qian & Law (1997) JFM 331

We, Re, B, R = 0.2, 14.8, 0.2, 120 μm
Large droplets